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$SU_L(4) \times U(1)$ model for electroweak unification and sterile neutrinos

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Abstract Some of the basic problems in neutrino physics, such as new energy scales, the enormous gap between the neutrino masses and the lightest charged fermion mass, and the possible existence of sterile neutrinos in the eV mass range are studied in the local gauge group $SU_L(4) \times U(1)$ for electroweak unification, which does not contain fermions with exotic electric charges. It is shown that the neutrino mass spectrum can be decoupled from that of the other fermions. The further normal seesaw mechanism for neutrinos, with right-handed neutrino Majorana masses of order $M \gg M_{\text{weak}}$ as well a new eV-scale can be accommodated. The eV-scale seesaw may manifest itself in experiments like the Liquid Scintillation Neutrino Detector (LSND) and MiniBooNE (MB) experimental results and future neutrino experiments.

In recent years, enormous progress has been made in neutrino physics, which also has relevance to many fields other than particle physics; in particular, nuclear physics, astrophysics and cosmology. This has been made possible by the quantum mechanical phenomena of interferometry, which provides a sensitive method to explore extremely small effects. This has resulted in the discovery of neutrino oscillations, which imply that they have tiny but finite masses; this goes against the prediction of the standard model of particle physics. Thus, they provide evidence for new physics which goes beyond the standard model. New physics requires a new energy scale beyond that provided by the standard model, but such a scale has not yet been pinned down. Thus, one needs to consider extensions of the electroweak group, which would provide a new scale between the electroweak and grand unification. The neutrinos may also provide an understanding of the origin of matter (baryogenesis) through leptogenesis. For this purpose, right-handed neutrinos, which are seesaw partners of the light neutrinos, with a mass scale of 10^{10}–10^{11} GeV or even in the TeV region may be needed. Furthermore, one sees an enormous gap between the neutrino masses, revealed by neutrino oscillations, and the lightest charged fermion mass (m_e) in contrast to that between m_e and m_t (the top quark mass), which is populated by charged leptons and quarks. Furthermore, $(m_\nu)_{\max}/m_e < 2 \times 10^{-6}$, which needs to be understood. This may be an indication of decoupling of the neutrino mass spectrum from other fermions. Moreover, while all neutrino data can be explained by flavor oscillations of three active neutrinos [1], the Liquid Scintillation Neutrino Detector (LSND) anomaly [2] stands out. This anomaly together with the MiniBooNE (MB) experiment [3, 4] may require at least two sterile neutrinos [5, 6] that mix with the active neutrinos. Another possibility is by a process with a decaying sterile neutrino, again in the eV range [7]. Their mass is in the range of electrovolts. The purpose of this paper is to make a modest attempt towards understanding some of the problems mentioned above.

We consider the extension of the electroweak group $SU_L(2) \times U(1)$ to $SU_L(4) \times U_X(1)$, as such an extension may answer some of the questions raised above, as we shall see. In particular, it is shown that, in addition to the normal seesaw mechanism for the neutrino masses, where the right-handed Majorana mass $\gg M_{\text{weak}}$, a new eV-scale can be accommodated. The latter may be a manifestation of the LSND and MB experiments. Furthermore, we show that the neutrino mass spectrum can be decoupled from that of the charged leptons.

If one restricts oneself to only $SU_L(4)$ [8], one can accommodate not only the known leptons nicely, but also a right-handed Majorana neutrino. However, in order to accommodate the quarks, the group has to be extended to $SU_L(4) \times U_X(1)$ [9–11]. In the original minimal version, where the leptons $(l^c, \nu_l^c, N_l, l)_R$ form an $SU(4)$ quartet, in order to cancel the anomalies, one has to have quarks with

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exotic electric charges: $\frac{4}{3}$ and $\frac{5}{3}$. In this paper we shall consider other versions, which do not involve quarks with exotic charges.[1]

The electric charge operator can, in general, be defined as a linear combination of diagonal generators of the group [$\widehat{I}$ being the unit matrix]

$$\begin{aligned} Q &= \frac{1}{2}\left[\lambda_3 + \frac{b}{\sqrt{3}}\lambda_8 + \frac{2c}{\sqrt{6}}\lambda_{15}\right] + \frac{Y_X}{2}\widehat{I} \\ &= \mathrm{diag}\left[\frac{1}{2}+\frac{b}{6}+\frac{c}{6}+\frac{Y_X}{2}, -\frac{1}{2}+\frac{b}{6}+\frac{c}{6}+\frac{Y_X}{2}, \right. \\ &\qquad \left. -\frac{2b}{6}+\frac{c}{6}+\frac{Y_X}{2}, -\frac{c}{2}+\frac{Y_X}{2}\right] \\ &= \frac{1}{2}(\lambda_3 + \widehat{Y}_1) + \frac{Y_X}{2}\widehat{I}, \end{aligned} \tag{1}$$

where Y_X is the hypercharge associated with U_X and

$$\widehat{Y}_1 = \mathrm{diag}\left[\frac{b+c}{3}, \frac{b+c}{3}, \frac{-2b+c}{3}, -c\right]. \tag{2}$$

Now

$$\frac{1}{e^2} = \frac{1}{g^2} + \frac{1}{g'^2},$$
$$\frac{1}{g'^2} = \frac{1}{g_1^2} + \frac{1}{g_X^2},$$

where in the $SU_L(4)$ limit

$$\begin{aligned} g_1^2 &= \frac{1}{C_1^2} g^2, \\ C_1^2 &= \frac{b^2+2c^2}{3}. \end{aligned} \tag{3}$$

This gives

$$\frac{1}{g'^2} = \frac{b^2+2c^2}{3g^2} + \frac{1}{g_X^2}.$$

Since $\frac{g'}{g} = \tan\theta_W$, one obtains

$$\frac{g_X^2}{g^2} = \frac{\sin^2\theta_W(m_X)}{1 - \frac{3+b^2+2c^2}{3}\sin^2\theta_W(m_X)}. \tag{4}$$

In the minimal version, $b=1$, $c=2$, and we have

$$\frac{g_X^2}{g^2} = \frac{\sin^2\theta_W(m_X)}{1-4\sin^2\theta_W(m_X)}, \tag{5}$$

and $Q = (1,0,0,-1) + \frac{Y_X}{2}(1,1,1,1)$, so that for leptons $Y_X = 0$, whereas, in order to accommodate quarks, we take $Y_X = -\frac{2}{3}$ for the first two generations of quarks and $Y_X = -\frac{1}{3}$ for the third generation. This is because in order to cancel the anomalies, one generation is to be treated differently from the other two. In this case, we have quarks with the exotic electric charges $-\frac{4}{3}$ and $-\frac{5}{3}$, respectively.

In order to accommodate known isospin doublets of left-handed quarks and leptons in the two upper components of 4 and 4*(or 4* and 4) representations of $SU(4)$ and to forbid exotic electrical charges, we must have $\frac{b+c}{6} = \pm\frac{1}{4}$, $\frac{-2b+c}{6} = -\frac{c}{2}$, so that $b = 2c = \pm 1$. We thus consider the version with $b = 2c = 1$ (the other choice is equivalent). This choice gives [$C_1^2 = \frac{1}{2}$]

$$\frac{g_X^2}{g^2} = \frac{\sin^2\theta_W(m_X)}{1-\frac{3}{2}\sin^2\theta_W(m_X)}. \tag{6}$$

A straightforward application of renormalization group equations gives [9] [$\sin^2\theta_W = \sin^2\theta_W(m_Z)$]

$$\begin{aligned} &1 - \left(1+C_1^2\right)\sin^2\theta_W - \frac{\alpha_X^{-1}(m_Z)}{\alpha^{-1}(m_Z)} \\ &\quad = 2\frac{\alpha(m_Z)}{4\pi}\left[-C_1^2\left(-\frac{22}{3}+\frac{4}{3}\frac{nf}{2}\right)+\frac{4}{3}\frac{nf}{2}C_1^2\right]\ln\frac{m_X}{m_Z} \\ &\quad = \frac{\alpha(m_Z)}{4\pi}\frac{44}{3}C_1^2\ln\frac{m_X}{m_Z}. \end{aligned}$$

For our case $C_1^2 = \frac{1}{2}$, and we obtain

$$1 - \frac{3}{2}\sin^2\theta_W - \frac{\alpha(m_Z)}{\alpha_X(m_Z)} = \frac{22}{3}\frac{\alpha(m_Z)}{4\pi}\ln\frac{m_X}{m_Z},$$

where $\sin^2\theta_W(m_Z) = 0.23122$ and $\alpha^{-1}(m_Z) = 128$. The unification scale m_X is not very sensitive to α_X. For $m_X = 10^3, 10^6, 10^{10}, 10^{16}$ GeV, $\alpha_X = 1.22\times10^{-2}, 1.28\times10^{-2}, 1.37\times10^{-2}, 1.54\times10^{-2}$, respectively. To put it in proper perspective, we note that the coupling $\alpha' = \frac{\alpha}{\cos^2\theta_W}$ associated with $U(1)$ of the Standard Model is $\simeq 1.30\alpha \simeq 1.02\times10^{-2}$. Before we give the anomaly free fermion content, we note that, for our choice $b = 2c = 1$,

$$Q = \mathrm{diag}\left[\frac{3}{4}+\frac{Y_X}{2}, -\frac{1}{4}+\frac{Y_X}{2}, -\frac{1}{4}+\frac{Y_X}{2}, -\frac{1}{4}+\frac{Y_X}{2}\right].$$

We can have two possibilities; they are given in Table 1.

The second alternative is very attractive, as it can naturally accommodate more than one right-handed neutrino per generation, some of which can be identified with sterile neutrinos when the $SU_L(4)\times U_X(1)$ symmetry is suitably broken in the lepton sector.

We first note that the $SU(4)$ lepton multiplet splits as follows under the subgroup $SU_L(2)\times U_{Y_1}(1)$ of $SU(4)$ as a

[1] For a model based on $SU_L(3)\times U_X(1)$ with fermions without exotic electric charges, see for example [12].

Table 1 Anomaly free fermion content (i is the generation index, a is the color index and c stands for charge conjugation)

$SU(4)$ quartet:		$SU(4)$ singlet:
I		
$F^l_{iL}=\begin{pmatrix}\nu_i\\ e_i^-\\ E_i^-\\ F_i^-\end{pmatrix}_{L,\,Y_X=-\frac{3}{2}}$	$i=1,2,3$	$F^l_{iR}\equiv\begin{pmatrix}N_i\ e_i^-\ E_i^-\ F_i^-\end{pmatrix}_R$ $Y_X:\ \ \begin{pmatrix}0\ -2\ -2\ -2\end{pmatrix}$
$F^q_{1L}=\begin{pmatrix}u_1^a\\ d_1^a\\ D_1^a\\ H_1^a\end{pmatrix}_{L,\,Y_X=-\frac{1}{6}}$		$F^q_{1R}\equiv\begin{pmatrix}u_1^a\ d_1^a\ D_1^a\ H_1^a\end{pmatrix}_R$ $Y_X:\ \ \begin{pmatrix}\frac{4}{3}\ -\frac{2}{3}\ -\frac{2}{3}\ -\frac{2}{3}\end{pmatrix}$
$F^q_{iR}=\begin{pmatrix}d_i^{ac}\\ u_i^{ac}\\ U_i^{ac}\\ T_i^{ac}\end{pmatrix}_{R,\,Y_X=-\frac{5}{6}}$	$i=2,3$	$F^q_{iR}\equiv\begin{pmatrix}d_i^{ac}\ u_i^{ac}\ U_i^{ac}\ T_i^{ac}\end{pmatrix}_R$ $Y_X:\ \ \begin{pmatrix}\frac{2}{3}\ -\frac{4}{3}\ -\frac{4}{3}\ -\frac{4}{3}\end{pmatrix}$
II		
$F^l_{1R}=\begin{pmatrix}e_i^c\\ {\nu_i}^c\\ N_i\\ N_i^s\end{pmatrix}_{R,\,Y_X=\frac{1}{2}}$		$e^c_{iL},\ Y_X=2$
$F^q_{1R}=\begin{pmatrix}d_1^{ac}\\ u_1^{ac}\\ U_1^{ac}\\ T_1^{ac}\end{pmatrix}_{R,\,Y_X=-\frac{5}{6}}$		$F^q_{1L}\equiv\begin{pmatrix}d_1^a\ u_1^a\ U_1^a\ T_1^{ac}\end{pmatrix}_L$ $Y_X:\begin{pmatrix}\frac{2}{3}\ -\frac{4}{3}\ -\frac{4}{3}\ -\frac{4}{3}\end{pmatrix}$
$F^q_{iL}=\begin{pmatrix}u_i^a\\ d_i^a\\ D_i^a\\ H_i^a\end{pmatrix}_{L,\,Y_X=-\frac{1}{6}}$	$i=2,3$	$F^q_{iR}\equiv\begin{pmatrix}u_i^a\ d_i^a\ D_i^a\ H_i^a\end{pmatrix}_R$ $Y_X:\ \ \begin{pmatrix}\frac{4}{3}\ -\frac{2}{3}\ -\frac{2}{3}\ -\frac{2}{3}\end{pmatrix}$

doublet:

$$\begin{pmatrix}e_i^c\\ {\nu_i}^c\end{pmatrix}_R, \qquad Y_1=\frac{1}{2},$$

and two singlets:

$$\begin{pmatrix}N_i & N_i^s\end{pmatrix}_R, \qquad Y_1=-\frac{1}{2}.$$

After breaking the group $SU(4)\times U_X(1)$ to $SU_L(2)\times U_Y(1)$ of the standard model, we have a doublet, $\binom{e_i^c}{\nu_i^c}_R$ with $Y=1$, a singlet, e^c_{iL} with $Y=2$ and two singlets, $(N_i \quad N_i^s)_R$ with $Y=0$. It is clear that the two extra neutrinos are decoupled from the group $SU_L(2)\times U_Y(1)$.

The interaction Lagrangian is given by (suppressing the generation index i)

$$\begin{aligned}
\mathcal{L}_I = &-\frac{g}{\sqrt{2}}\big[\overline{e}^c_R\gamma^\mu\nu^c_R W^-_\mu+\text{h.c.}\big]\\
&-\frac{g}{2}\bigg[\overline{e}^c_R\gamma^\mu e^c_R\bigg(W_{3\mu}+\frac{g_1}{2g}B_{1\mu}+\frac{g_X}{2g}V_\mu\bigg)\\
&+\overline{e}^c_L\gamma^\mu e^c_L\bigg(2\frac{g_X}{g}V_\mu\bigg)\\
&+\overline{\nu}^c_R\gamma^\mu\nu^c_R\bigg(-W_{3\mu}+\frac{g_1}{2g}B_{1\mu}+\frac{g_X}{2g}V_\mu\bigg)\\
&+\overline{N}_R\gamma^\mu N_R\bigg(U_{3\mu}-\frac{1}{2}\frac{g_1}{g}B_{1\mu}+\frac{g_X}{2g}V_\mu\bigg)\\
&+\overline{N}^s_R\gamma^\mu N^s_R\bigg(-U_{3\mu}-\frac{1}{2}\frac{g_1}{g}B_{1\mu}+\frac{g_X}{2g}V_\mu\bigg)\bigg]\\
&-\frac{g}{\sqrt{2}}\big[(\overline{e}^c_R\gamma^\mu N_{eR}X^-_\mu+\overline{\nu}^c_R\gamma^\mu N_R X^0_\mu+\overline{e}^c_R\gamma^\mu N^s_R Y^-_\mu\\
&+\overline{\nu}^c_R\gamma^\mu N^s_R Y^0_\mu+\overline{N}_R\gamma^\mu N^s_R U)+\text{h.c.}\big],
\end{aligned}\tag{7}$$

where the vector boson $B_{1\mu}=\sqrt{\frac{2}{3}}W_{8\mu}+\sqrt{\frac{1}{3}}W_{15\mu}$ is coupled to $U_{Y_1}(1)$ and $U_{3\mu}=-\sqrt{\frac{1}{3}}W_{8\mu}+\sqrt{\frac{2}{3}}W_{15\mu}$. Note that in the symmetry limit, we have $g_1=\sqrt{2}g$. Furthermore, we note that the vector boson B_μ corresponding to $U_Y(1)$ is given by

$$\frac{B_\mu}{g'}=\frac{B_{1\mu}}{g_1}+\frac{V_\mu}{g_X}.\tag{8}$$

Thus,

$$\begin{aligned}
A_\mu &= \frac{e}{g}W_{3\mu}+\frac{e}{g_1}B_{1\mu}+\frac{e}{g_X}V_\mu\\
&= \frac{e}{g}W_{3\mu}+\frac{e}{g'}B_\mu,\\
Z_\mu &= \frac{e}{g'}W_{3\mu}-\frac{e}{g}B_\mu.
\end{aligned}\tag{9}$$

There are two more vector bosons, which we define as follows:

$$\begin{aligned}
Z'_\mu &= -\frac{g_1}{g}B_{1\mu}+\frac{g_X}{g}V_\mu,\\
Z''_\mu &= U_{3\mu}.
\end{aligned}\tag{10}$$

Hence, rewriting the interaction Lagrangian in terms of vector bosons A_μ, Z_μ, Z'_μ and Z''_μ, we have

$$\begin{aligned}
\mathcal{L}_I^{\text{neutral}} = &-g\sin\theta\big[\overline{e}^c_R\gamma^\mu e^c_R+\overline{e}^c_L\gamma^\mu e^c_L\big]A_\mu\\
&-\frac{g}{2\cos\theta_W}\big[\big(\overline{e}^c_R\gamma^\mu e^c_R-\overline{\nu}^c_R\gamma^\mu\nu^c_R\big)\\
&-2\sin^2\theta_W\big(\overline{e}^c_R\gamma^\mu e^c_R+\overline{e}^c_L\gamma^\mu e^c_L\big)\big]Z_\mu\\
&-\frac{1}{2}g\bigg[\frac{1}{2}\big(\overline{e}^c_R\gamma^\mu e^c_R+\overline{\nu}^c_R\gamma^\mu\nu^c_R+\overline{N}_R\gamma^\mu N_R\\
&+\overline{N}^s_R\gamma^\mu N^s_R\big)
\end{aligned}$$

$$-\frac{g^2}{g_1^2}\tan^2\theta_W\big(\overline{e}^c_R\gamma^\mu e^c_R-2\overline{e}^c_L\gamma^\mu e^c_L+\overline{\nu}^c_R\gamma^\mu\nu^c_R\big)\Big]Z'_\mu-\frac{1}{2}g\big[\overline{N}_R\gamma^\mu N_R-\overline{N}^s_R\gamma^\mu N^s_R\big]Z''_\mu. \tag{11}$$

From (7) and (11), it is clear that the doublet $\binom{N_i}{N_i^s}_R$ belongs to the fundamental representation of the U-spin $SU(2)$ subgroup of $SU(4)$ (the other subgroup being $SU_L(2)$) with vector bosons $U,\overline{U},U_{3\mu},\equiv Z''_\mu$ belonging to the adjoint representation of this group. To break the symmetry and at the same time give Dirac masses to fermions which have both left-handed and right-handed components viz. charged leptons and quarks, the minimally required Higgs bosons are given in Table 2.

The following comments are in order: ρ and χ correspond to the Higgs field

$$\phi=\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix},\qquad \langle\phi\rangle_0=\begin{pmatrix}0\\ \frac{v}{\sqrt{2}}\end{pmatrix},$$

$$\widetilde{\phi}=\mathrm{i}\tau\overline{\phi}=\begin{pmatrix}\phi_0^*\\ -\phi^-\end{pmatrix},\qquad \langle\widetilde{\phi}\rangle=\begin{pmatrix}\frac{v}{\sqrt{2}}\\ 0\end{pmatrix}.$$

Thus, $\langle\rho\rangle$ and $\langle\chi\rangle$ give masses to the vector bosons of the standard model gauge group and masses to the charged leptons and quarks of the standard model. For charged leptons, only $\langle\chi\rangle$ contributes. The other Higgs bosons are needed to break the group $SU(4)\times U(1)$ so as to give superheavy masses to the extra vector bosons and extra quarks outside the standard model.

Table 2 Higgs quartets

$Y_X=-\frac{3}{2}$		
$\chi=\begin{pmatrix}\chi^0\\ \chi^-\\ \chi'^-\\ \chi''^-\end{pmatrix}$	$\langle\chi\rangle=\begin{pmatrix}u\\0\\0\\0\end{pmatrix}$	
$Y_X=+\frac{1}{2}$		
$\rho=\begin{pmatrix}\rho^+\\ \rho^0\\ \rho'^0\\ \rho''^0\end{pmatrix}$	$\eta=\begin{pmatrix}\eta^+\\ \eta^0\\ \eta'^0\\ \eta''^0\end{pmatrix}$	$\xi=\begin{pmatrix}\xi^+\\ \xi^0\\ \xi'^0\\ \xi''^0\end{pmatrix}$
$\langle\rho\rangle=\begin{pmatrix}0\\u'\\0\\0\end{pmatrix}$	$\langle\eta\rangle=\begin{pmatrix}0\\0\\V\\0\end{pmatrix}$	$\langle\xi\rangle=\begin{pmatrix}0\\0\\0\\V'\end{pmatrix}$

With the symmetry breaking pattern discussed above, the mass Lagrangian for vector bosons is given by

$$\begin{aligned}\mathcal{L}^W_{\text{mass}}=&\frac{1}{2}g^2u^2\Big[2W^+W^-+\Big(\frac{1}{\cos\theta_W}Z+\frac{1}{2}\Big(1-6\frac{g^2}{g_1^2}\tan^2\theta_W\Big)Z'\Big)^2+2X^+X^-+2Y^+Y^-\Big]\\&+\frac{1}{2}g^2u'^2\Big[2W^+W^-+\Big(\frac{1}{\cos\theta_W}Z+\frac{1}{2}\Big(1-2\frac{g^2}{g_1}\tan^2\theta_W\Big)Z'\Big)^2+2\overline{X}^0X^0+2\overline{Y}^0Y^0\Big]\\&+\frac{1}{2}gV^2\Big[2X^+X^-+2\overline{X}^0X^0+2U\overline{U}+\Big(\frac{1}{2}Z'+Z''\Big)^2\Big]\\&+\frac{1}{2}gV'^2\Big[2Y^+Y^-+2\overline{Y}^0Y^0+2U\overline{U}+\Big(\frac{1}{2}Z'-Z''\Big)^2\Big].\end{aligned}\tag{12}$$

Since $V\approx V'\gg u\approx u'$, therefore neglecting the terms of order $\frac{u^2}{V^2}$, we can write

$$\begin{aligned}\mathcal{L}^W_{\text{mass}}\approx&\frac{1}{2}g^2\Big[(u^2+u'^2)\Big(2W^+W^-+\frac{1}{\cos^2\theta_W}Z^2\Big)\Big]\\&+\frac{1}{2}g^2\Big[V^2\big(2X^+X^-+2X^0\overline{X}^0+2U\overline{U}\big)+\Big(\frac{1}{2}Z'+Z''\Big)^2+V'^2\Big(\Big(\frac{1}{2}Z'-Z''\Big)^2+2Y^+Y^-+2Y^0\overline{Y}^0+2U\overline{U}\Big)\Big].\end{aligned}\tag{13}$$

This gives the gauge boson masses:

$$m_W^2=\frac{1}{2}g^2(u^2+u'^2)=\frac{m_Z^2}{\cos^2\theta_W},$$

$$m_X^2=\frac{1}{2}g^2V^2,\qquad m_Y^2=\frac{1}{2}g^2V'^2,\qquad m_U^2=m_X^2+m_Y^2,$$

$$m^2_{Z'Z''}=\frac{1}{2}g^2\begin{pmatrix}\frac{V^2+V'^2}{4} & \frac{V^2-V'^2}{2}\\ \frac{V^2-V'^2}{2} & V^2+V'^2\end{pmatrix}.$$

So far we have introduced essentially two energy scales, represented by the vacuum expectation values u ($\simeq u'$) and V

($\simeq V'$); the former corresponds to the SM-scale ($\simeq$175 GeV) and the latter, although not fixed much higher—an interesting one would be an intermediate energy scale (between SM and GUT, $\simeq 10^{10}$ GeV).

The Lagrangian (11) explicitly shows the decoupling of extra leptons from the standard model. Only the connection with the standard model is through the mixing of Z with Z' (involving terms of order $\frac{u^2}{V^2}$), when the symmetry is broken to give masses to the gauge bosons. Since the extra bosons other than the standard bosons are very heavy, the mixing terms only give a negligible contribution to the standard model observables. Similarly, Z' being very heavy, its contribution to the standard model observables is also negligible.

We now discuss the $SU_L(4)\times U_X(1)$ Yukawa interaction in the charged lepton sector, which is

$$H_Y = h_{ij}\left(\overline{F}^c_{iR}\chi_j e^c_{iL}\right) + \text{h.c.},$$

so that

$$M_l = h_{ij}(\overline{e}_{iL}u_j e_{iR}) + \text{h.c.} \tag{14}$$

For simplicity we may take $u_e = u_\mu = u_\tau = u$. It is important to note that we have no new charged leptons, other than those of the standard model. As far as new quarks are concerned, their masses will be determined essentially by $h_{iQ}V$, h_{iQ} being the corresponding Yukawa coupling constant. Q stands for U, T, D and H quarks. If the Yukawa couplings are of order unity, the masses of the new quarks will be of the same order as those of the vector bosons X etc.

The above scalars do not give masses to neutrinos. In a way this is a great advantage, as neutrinos have a mass spectrum completely different from the other fermions.

To give Majorana masses to the neutrinos, we introduce the Higgs scalars $S_{\alpha\beta}$ [α,β are $SU(4)$ indices] belonging to the symmetric representation 10 of $SU(4)$ with $Y_X = 1$. The electric charge matrix for this representation is

$$\widehat{Q} = \begin{pmatrix} 2 & 1 & 1 & 1\\ 1 & 0 & 0 & 0\\ 1 & 0 & 0 & 0\\ 1 & 0 & 0 & 0\end{pmatrix}.$$

Thus, charged leptons cannot get any masses from the Higgs $S_{\alpha\beta}$ nor the quarks. In order to make N heavy and N^s light ($\sim$eV) we introduce

$$\overline{S}_{\alpha\beta},\quad \overline{S}'_{\alpha\beta},\quad \text{and}\quad \overline{S}''_{\alpha\beta}\quad \text{with } Y_X = -1,$$

with their expectation values in Table 3, where $\kappa_s, \kappa'_s \ll \kappa' \ll \kappa_R$. We wish to remark that we have selected three different Higgs scalars $S_{\beta\alpha}$ with the same $Y_X = -1$, just as we selected three Higgs quartets ρ, η, ξ with $Y_X = -\frac{1}{2}$, for the reason that, with one $S_{\beta\alpha}$, the vacuum expectation values $\kappa', \kappa_R, \kappa_s, \kappa'_s$ as different components of the same Higgs scalar would have been critically aligned. They, now belonging to three different Higgs scalars, are of course very hierarchical to accommodate different energy scales. However, the hierarchy problem is there for models which go beyond the standard model and has no easy solution as is well known. These additional Higgs bosons give an extra contribution to $\mathcal{L}^W_{\text{mass}}$ in (13) [only κ_R is important]

$$\frac{1}{2}g^2\kappa_R^2\left[X^+X^- + \overline{X}^0X^0 + 2\left(\frac{1}{2}Z' + Z''\right)^2\right],$$

so that only the X and Z' and Z'' gauge bosons get an extra contribution, giving a further splitting between these and the Y and U bosons.

Table 3 Vacuum expectation values of Higgs belonging to the 10 representation of $SU(4)$

$$\langle\overline{S}'_{\alpha\beta}\rangle = \begin{pmatrix} 0 & 0 & 0 & 0\\ 0 & 0 & \kappa' & 0\\ 0 & \kappa' & 0 & 0\\ 0 & 0 & 0 & 0\end{pmatrix} \qquad \langle\overline{S}''_{\alpha\beta}\rangle = \begin{pmatrix} 0 & 0 & 0 & 0\\ 0 & 0 & 0 & \kappa'_s\\ 0 & 0 & 0 & 0\\ 0 & \kappa'_s & 0 & \kappa_s\end{pmatrix}$$

$$\langle\overline{S}_{\alpha\beta}\rangle = \begin{pmatrix} 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & \kappa_R & 0\\ 0 & 0 & 0 & 0\end{pmatrix}$$

The Yukawa couplings of these scalars to leptons are

$$\sum_b f^b_{ij}F^T_{i\alpha}C^{-1}F_{j\beta}\overline{S}^b_{\beta\alpha}, \tag{15}$$

where b is for no prime, a prime and a double prime. Then the neutrino mass matrix is given by

$$\begin{aligned} M_N &= f_{ij}\left(\kappa_R N^T_i C^{-1}N_j\right) + f'_{ij}\kappa'\overline{\nu}_i N_j\\ &\quad + f''_{ij}\left(\kappa_s N^{sT}_i C^{-1}N^s_j + \kappa'_s\overline{\nu}_i N^s_j\right) + \text{h.c.} \end{aligned} \tag{16}$$

Equation (16) gives a 9×9 neutrino mass matrix in the weak eigenstate basis:

$$M_{\nu_N} = \begin{pmatrix} 0 & m_D & m^s_D\\ m^T_D & M & 0\\ m^{sT}_D & 0 & m^s\end{pmatrix}, \tag{17}$$

where $(m_D)_{ij} = f'_{ij}\kappa'$, $(M)_{ij} = f_{ij}\kappa_R$, $(m^s_D)_{ij} = f''_{ij}\kappa'_s$, $(m^s)_{ij} = f''_{ij}\kappa_s$.

Since κ_s, $\kappa'_s \ll \kappa' \ll \kappa_R$, in the limit $m^s_D, m^s \to 0$, the diagonalization gives

$$M_{\nu_N} = \begin{pmatrix} A & 0 & 0\\ 0 & M & 0\\ 0 & 0 & 0\end{pmatrix}, \tag{18}$$

where $A = -m_D M^{-1} m_D^T$. However, by introducing a unitary matrix U:

$$U = \begin{pmatrix} 1 & b & 0 \\ -b^T & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad b = -m_D M^{-1}, \qquad b^T = -M^{-1} m_D^T \tag{19}$$

we obtain the mass matrix

$$\begin{aligned} U M_{\nu_N} U^T &= \begin{pmatrix} A & 0 & m_D^s \\ 0 & M & 0 \\ m_D^{sT} & 0 & m_s \end{pmatrix} + O\left(\frac{1}{M^2}\right) \\ &= M \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ &\quad + \begin{pmatrix} 0 & 0 & m_D^s \\ 0 & 0 & 0 \\ m_D^{sT} & 0 & m_s \end{pmatrix} + O\left(\frac{1}{M^2}\right). \end{aligned} \tag{20}$$

This gives the effective 6×6 light neutrino mass matrix

$$M_\nu = \begin{pmatrix} -m_D M^{-1} m_D^T & m_D^s \\ m_D^{sT} & m_s \end{pmatrix}. \tag{21}$$

The phenomenology of this 6×6 matrix already exists in the literature [4,5,12,13]. In (21) $-m_D M^{-1} m_D^T$ gives the normal seesaw mechanism. Here $M \gg m_D$, and it may be of order 10^{10}–10^{12} GeV, a scale which may be relevant for thermal, non-degenerate leptogenesis. Now a word about energy scales: κ_R is of the order of 10^{10} GeV, while κ' is of the order of 1 GeV, so that if all Yukawa coupling constants in (16) are of order unity, the active neutrino mass must be of order 0.1 eV to satisfy the constraint from the WMAP data, $\sum m_{\nu_i} < (0.4\text{–}0.7)$ eV, while atmospheric data give $m_\nu > 5 \times 10^{-2}$ eV. κ_s is of the order of a few eV; sterile neutrinos are to be relevant for "short" baseline oscillation searches. Finally, in order to have small active–sterial mixing (∼0.1) [12, 13], we have $\kappa_s' \simeq 0.1\kappa_s$. For other approaches to the possible existence of sterile neutrinos in the eV-scale range within the framework of higher order gauge groups beyond the standard model, see references [14–16].

In summary, we have shown that (i) intermediate mass scales between the electroweak mass scale and that of grand unification can be accommodated, which is relevant for leptogenesis; (ii) one can naturally accommodate more than one right-handed neutrino per generation, some of which can be identified with sterile neutrinos that mix with active ones; (iii) the neutrinos mass spectrum is decoupled from other fermions; (iv) we have both the normal seesaw mechanism with a right-handed neutrino Majorana mass $M \gg M_{\text{weak}}$, which may be relevant for leptogenesis and an eV-scale which may manifest itself in "short" baseline oscillation searches neutrino experiments.

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